AP Calculus Final Exam Review

Limits and Continuity

1. Evaluate the following limits:

a) $\lim_{x\to 2}\frac{x^2+7x+3}{x-7}$ b) $\lim_{x\to 3}\frac{x^2-x-6}{x-3}$ c) $\lim_{x\to 6}\frac{\sqrt{3+x}-3}{x-6}$ d) $\lim_{m\to 10}\frac{\frac{2}{m-4}-\frac{1}{3}}{m-10}$

e) $\lim_{x\to\infty}\frac{5-6x}{2x+3}$ f) $\lim_{x\to\infty}\frac{3x^2}{x^3-4x}$ g) $\lim_{x\to-\infty}\frac{x^2+5x}{x-5}$ h) $\lim_{x\to-\infty}\frac{\sqrt{4x^2-x-8}}{x+9}$

i) $\lim_{x\to-4^-}\frac{x+6}{x+4}$ j) $\lim_{x\to 3^-}\frac{x}{(x-3)^2}$ k) $\lim_{x\to 1}f(x)$ if $f(x) = \begin{Bmatrix} x^3 - 2x^2, if\ x \geq 1 \\ \frac{x+2}{x-2},\ \ if\ x < 1 \end{Bmatrix}$

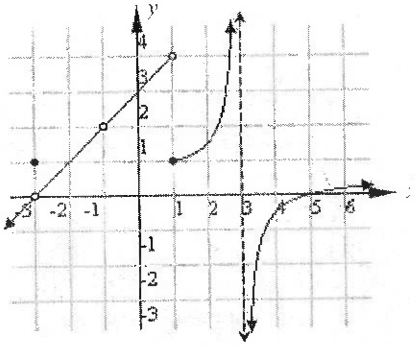
2. Find the value of *k* if the function $f(x) = \begin{Bmatrix} x^2 + x, if\ x < -4 \\ kx + 10\,, if\ x \geq -4 \end{Bmatrix}$ is to be continuous.

3. Using the test for continuity at a point, explain why each function is discontinuous at the given x-value. Classify the discontinuity.

a) $f(x) = \frac{7}{x-4}, at\ x = 4$

b) $f(x) = \begin{Bmatrix} x^2 + 2x, if\ x \leq 3 \\ 4x + 5\,, if\ x > 3 \end{Bmatrix}, at\ x = 3$

4. Given the following diagram, state the x-coordinates of the points of discontinuity, classify the discontinuity, and explain where it breaks down in the three part definition of continuity at a point.



5. Find the horizontal and vertical asymptotes for the function $f(x) = \frac{2x^2-1}{4-x^2}$

Answers to Limits and Continuity:

1. a) $\frac{-21}{5}$ b) 5 c) $\frac{1}{6}$ d) $\frac{-1}{18}$ e) -3 f) 0 g) -∞ h) -2 i) -∞ j) ∞ k) DNE 2. $k = \frac{-1}{2}$ 3. a) infinite discontinuity, breaks down at i) f(4) DNE b) jumps discontinuity at x = 3, breaks down at ii) limit as x approaches 3 DNE 4. x = -3 removable, breaks down at iii), limit at -3 doesn't equal value of function at -3, x = -1 removable, breaks down at i), f(-1) DNE, x = 1 jump, breaks down at ii) limit does not exist. x = 3 infinite, breaks down at i), f(3) DNE.

5. HA y=-2 VA x=2, x=-2

Derivatives

1. Find the first derivatives of the following functions, and simplify:

(a) $f(x) = (2x+3)^2$

(b) $f(x) = (x^3 + x)(8 - x)$

(c) $f(x) = \left(\frac{2x}{x+2}\right)^{-2}$

(d) $y = \frac{\sqrt{x+3}}{x-2}$

(e) $f(x) = (x+3)e^{3x}$

(f) $y = -6\sin(3x^2)$

(g) $f(x) = \ln\left(\frac{x^2}{x-2}\right)$

(h) $y = x^2 \tan 3x$

(i) $y = \cos^3(x^2 + 2)$

(j) $y = 8^{x^3}$

2. Use implicit differentiation to find the first and second derivative of $y^2 + 3y - x = 4$.

3. Find the equation of the tangent line to the function $f(x) = x^3 + 2x - 7$ at x = 2. Write your answer in slope intercept form.

4. Find the coordinates of the point on $f(x) = x^2 - 5x + 1$ where the tangent line is parallel to $5x - y = 10$.

5. Find the equation of the normal to the function $f(x) = (4x+3)^2$ at the point (1,49).

Answers to Derivatives:

1. a) $4(2x + 3)$ b) $-2(2x^3 - 12x^2 + x - 4)$ c) $\frac{-(x+2)}{x^3}$ d) $\frac{-(x+8)}{2(x-2)^2\sqrt{x+3}}$ e) $e^{3x}(3x + 10)$

f) $-36x\cos(3x^2)$ g) $\frac{x-4}{x(x-2)}$ h) $x(3x\sec^2 3x + 2\tan 3x)$ i) $-6x\sin(x^2 + 2)\cos^2(x^2 + 2)$

j) $8^{x^3}3x^2\ln 8$ 2. $\frac{dy}{dx} = \frac{1}{2y+3}, \frac{d^2y}{dx^2} = \frac{-2}{(2y+3)^3}$ 3. $y = 14x - 23$ 4. (5,1) 5. $x + 56y = 2745$

Application of Derivatives

1. Find the critical numbers, if any, for each of the following functions.

a) $f(x)=x^2+6x-8$

b) $f(x)=5x^{1/5}$

c) $f(x)=\dfrac{x}{(x+1)^2}$

2. Without drawing the graph, determine the absolute extrema for the function on the given interval.

a) $f(x)=4-x^2$; $[-1,2]$

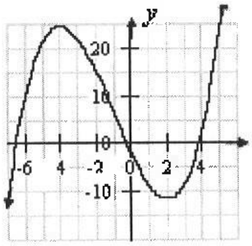
b) $f(x)=\dfrac{27}{x^2+9}$; $[-2,3]$

c) $f(x)=x\sqrt{3-x}$; $[-1,3]$

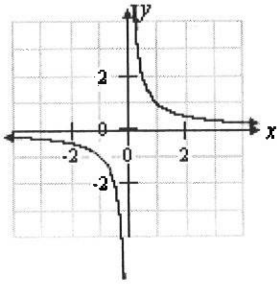
3. Find the value of "c" that satisfies the Mean Value Theorem for the function $f(x) = x^3 + 2x^2 - x$ over the interval [-1,2].

4. By performing a sign analysis on $f'(x)$, determine the open interval(s) in which each of the following functions is increasing or decreasing. Check your answers by examining the graph provided.

a. $f(x)=\dfrac{1}{3}x^3+x^2-8x-2$



b. $f(x)=\dfrac{1}{x}$



5. Find the intervals in which the function is increasing or decreasing. Find the coordinates of any relative extrema. Use the first derivative test. Verify with a graphing calculator.

a. $f(x) = (x+2)^3$

b. $f(x) = x^3 + 6x^2 - 15x - 90$

6. Find $f''(x)$. On the basis of a sign analysis of $f''(x)$, determine the intervals in which the function is concave up or concave down. Confirm with a graphing calculator.

a. $f(x) = x^3 - 3x^2$

b. $f(x) = x^4 + 6x^3$

c. $f(x) = \frac{4}{x^2+3}$

7. For the function find the following.
(a) a sign analysis of $f'(x)$
(b) the open intervals on which $f(x)$ is increasing and/or decreasing.
(c) the relative extrema
(d) a sign analysis of $f''(x)$
(e) the intervals on which $f(x)$ is concave up and concave down.
(f) the coordinates of any inflection points.
(g) the x and y intercepts.
(h) sketch the graph

$f(x) = x^3 + 3x^2 - 24x$

8. Two numbers have a sum of 4. How must they be chosen in order to maximize their product? What is that maximum product?

9. A rectangular box-shaped garbage can with a square base and an open top is to be constructed using exactly 2700 cm^2 of material. Find the dimensions of the box that will provide the greatest possible volume.

10. We have a piece of cardboard that is 14 inches by 10 inches and we're going to cut out square corners and fold up the sides to form a box. Determine the length of the cut that will give a maximum volume.

11. Water is being poured into a conical vase at a rate of $18 \text{ cm}^3/\text{s}$. The diameter of the cone is 30 cm and the height of the cone is 25 cm. At what rate is the water level rising when the water's depth is 20 cm?

12. A cylindrical tank has a radius of 3m and a depth of 10m. It is being filled at the rate of 5 m^3/min.

13. A plane flying north travels of LeBoldus at 12:00 pm and continues to travel north at 100km/h. Another plane flies over LeBoldus at 1:00pm travelling east at 85 km/h. At what rate is the distance between the planes changing at 3:00 pm? (Assume the two planes fly at a constant speed for the entire time).

14. A particle moves along the *x*-axis so that its position in metres after *t* seconds is given by the function: $s(t) = 2t^3 - 7t^2 + 3t$

   (a) Where is the ball after 3 seconds?

   (b) Find the velocity function.

   (c) Find the instantaneous velocity of the ball after 2 seconds.

   (d) Find the average velocity of the ball from 4 seconds to 6 seconds.

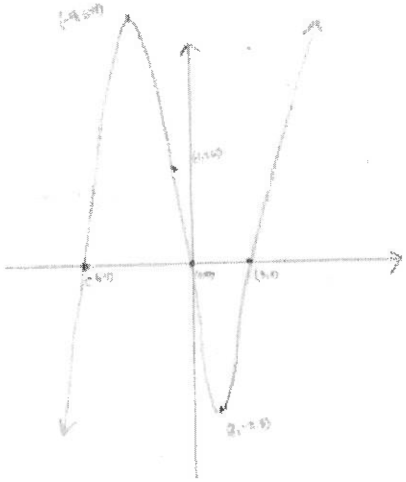
   (e) Find the acceleration function.

   (f) What is the instantaneous acceleration at 10 seconds?

   (g) What is the instantaneous velocity of the ball when the acceleration is 70 m/s^2?

Answers to Application of Derivatives:

1.a) $x = -3$ b) $x = 0$ c) $x = 1$ *and* $x = -1$ 2.a) abs max of 4, abs min of 0 b) abs max 3, abs min of $\frac{3}{2}$
c) abs max of 2, abs min of -2, 3. $c = 0.786$ 4.a) $inc(-\infty,-4) \cup (2,\infty)$, $dec\ (-4,2)$
b) $dec(-\infty,0) \cup (0,\infty)$ 5.a) $inc(-\infty,-2) \cup (-2,\infty)$, no relative extrema
b) $inc(-\infty,-5) \cup (1,\infty), dec(-5,1), rel\max f(-5) = 10, rel\min f(1) = -98$
6.a) $CU(1,\infty)\ and\ CD(-\infty,1)$ b) $CU(-\infty,-3) \cup (0,\infty) and\ CD(-3,0)$
c) $CU(-\infty,-1) \cup (1,\infty) and\ CD(-1,1)$ 7.b) $INC(-\infty,-4) \cup (2,\infty) and\ DEC(-4,2)$
c) $rel\max f(-4) = 80\ and\ rel\min f(2) = -28$ e) $CD(-\infty,-1) and\ CU\ (-1,\infty)$ f) $IP\ f(-1) = 26$
g) $y-\ int\ (0,0)\ and\ x-\ int\ (0,0), (-6.623,0), (3.623,0)$ h)



8. The numbers are 2 and 2 and the max product is 4.
9. The dimensionos are 30cm by 30cm by 15 cm.
10. The length of the cut is 1.918 inches.
11. The water klevel is rising at a rate of 0.040 cm/s
12. The tank level is rising at a rate of 0.177 m/min
13. The distance between the planes is increasing at a rate of 128.908 km/hr.
14. a) $s(3) = 0\ m$ b) $v(t) = 6t^2 - 14t + 3$
c) $v(2) = -1m/s$ d) $ave\ vel = 85m/s$
e) $a(t) = 12t - 14$ f) $a(10) = 106m/s^2$
g) $v(7) = 199m/s$

Definitie Integrals and Indefinite Integrals

1. Select values of $f(x)$ are shown in the table below over the interval [1,15]. Use a left end Riemann Sum with the six sub intervals in the chart to approximate $\int_1^{15} f(x)dx$.

X	1	3	5	8	9	13	15
f(x)	5	5.4	6.2	8.1	8.9	12.9	15.2

2. The rate at which water flows out of a pipe, in litres per hour, over a 24 hour period is given in the table below.

Time (h)	0	3	6	9	12	15	18	21	24
Rate (L/hr)	8.4	9.3	7.6	6.2	9.1	10.4	11.3	7.5	6.4

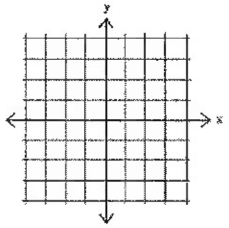
a) Use a midpoint Riemann sum with 4 subdivisions of equal width to approximate $\int_0^{24} R(t)dt$.

b) Using correct units, explain the meaning of the answer in part a)

c) Approximate $R'(t)$ at $t = 13$ h. **Include units in your answer.**

d) Evaluate $\frac{1}{24}\int_0^{24} R(t)dt$. Using correct units, explaing the meaning of this answer in the context of the problem.

3. Using a graph of the function $f(x) = \sqrt{9 - x^2}$ calculate the value of $\int_{-3}^{3} \sqrt{9 - x^2}dx$.



4. Evaluate the following definite integrals.

a) $\int_0^4 x^3 dx$

b) $\int_{-2}^{3} (x+1)^2 dx$

c) $\int_{\pi/2}^{\pi} \sin x dx$

d) $\int_3^5 \frac{1}{x^2}dx$

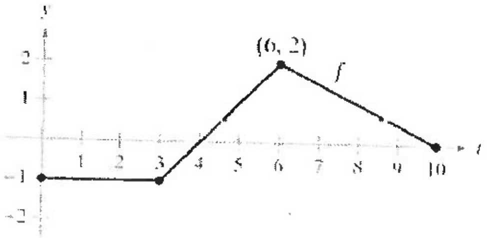
e) $\int_2^5 \frac{1}{x+3}dx$

f) $\int_{\ln 2}^{\ln 5} e^x dx$

g) $\int_{-5}^{5} (5x^4 + 6x)dx$

h) $\int_{1}^{9} \sqrt{x}\, dx$

5. Given that $g(x) = \int_0^x f(t)dt$, find the following:

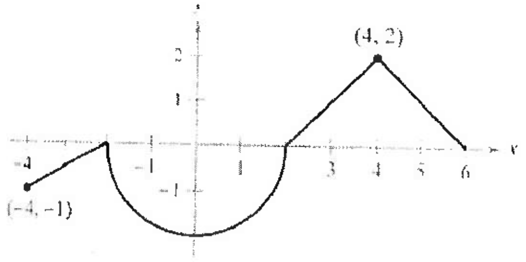


a) $g(4)$ b) $g(6)$ c) $g'(6)$ d) $g''(8)$

e) At what x coordinates does $g(x)$ have inflection points. Justify.

f) Find the equation of the tangent line to function g at the coordinate $x = 6$.

6. Given the graph of $f'(x)$ below over the closed interval [-4,6] and that $f(-2) = 4$ find the following:



a) $f(-4)$ b) $f(4)$

c) $f(6)$ d) Find the absolute minimum over the interval [-4,6]

e) Find all inflection points of $f(x)$ and justify.

f) Find the interval(s) where $f(x)$ is both decreasing and concave down. Justify.

7. Perform each of the following integrations.

a) $\int 1-x^{-2}+x^{-3}-x^{-4}dx$ b) $\int 6dx$ c. $\int \frac{4}{x}dx$ d) $\int -2x^{-3/2}dx$ e) $\int e^{11x}dx$ f) $\int (6x-\sin x+\cos x)dx$

g) $\int (x+1)^2 dx$ h) $\int \frac{2x-x^2}{\sqrt{x}}dx$ i) $\int \cos(3x)\,dx$

8. Use u substitution to integrate the following:

a) $\int (3x-8)^6 dx$

b) $\int x^2(2x^3+3)^{14}dx$

c) $\int x^2 \cos x^3 dx$

d) $\int (x+5)e^{x^2+10x}dx$

e) $\int \sqrt{e^x}dx$

<u>**Answers to Definite and Indefinite Integrals**</u>

1. 108.9 2. 200.4 litres b) this represents the amount of litres that flowed through the pipe from t=0h to t=24h. c) 0.4333 $litres/hr^2$ d) 8.350 litres/hour this represents the average rate flow of the water from t=0 to t=24 hours. 3. $\frac{9\pi}{2}$ 4.a) 64 b) $\frac{65}{3}$ c) 1 d) $\frac{2}{15}$ e) $\ln(\frac{8}{5})$ f) 3 g) 6250 h) $\frac{52}{3}$

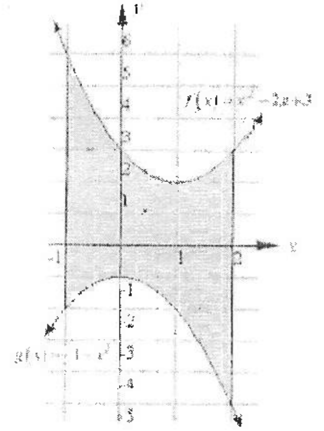
5.a) $\frac{-7}{2}$ b) $\frac{-3}{2}$ c) 2 d) -1/2 e) 6 because g' changes from inc to dec. f) $y = 2x - \frac{27}{2}$ 6. a) 5 b) $6-2\pi$ c) $8-2\pi$ d) $4-2\pi$ e) $x = -2{,}4$ because f' changes from inc to dec., $x = 0$ because f' changes from dec to inc f) $(-2{,}0)$ because f'<0 and f' is decreasing

7.a) $x + x^{-1} - \frac{x^{-2}}{2} + \frac{x^{-3}}{3} + C$b) $6x + C$ c) $4ln|x| + C$ d) $4x^{\frac{-1}{2}} + C$ e) $\frac{e^{11x}}{11} + C$ f) $3x^2 + cosx + sinx + C$ g) $\frac{x^3}{3} + x^2 + x + C$ h) $\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$ i) $\frac{1}{3}\sin(3x) + C$ 8.a) $\frac{1}{21}(3x-8)^7 + C$

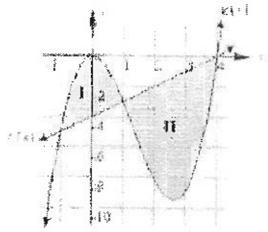
b) $\frac{1}{90}(2x^3+3)^{15} + C$ c) $\frac{1}{3}sinx^3 + C$ d) $\frac{1}{2}e^{x^2+10x} + C$ e) $2e^{\frac{1}{2}x} + C$

Applications of Integration

1. Find the area of the region bounded by $f(x) = x^2 - 2x + 3$ and $g(x) = -1 - x^2$ and the vertical lines $x = -1$ and $x = 2$.



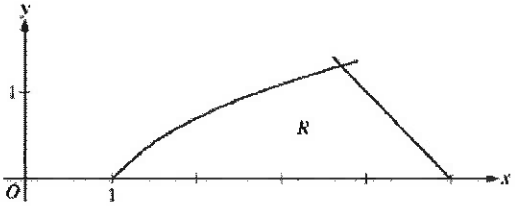
2. Find the area bounded by the curves $f(x) = x - 4$ and $g(x) = x^3 - 4x^2$.



3. Find the value of m such that $\int_m^4 4x dx = -18$

4. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.



b) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

Answers to Applications of Integration

1. 15 2. $\frac{253}{12}$ 3. 5, -5 4. a) 2.986 $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986$ b)

AP Calculus Final Exam Review

Limits and Continuity

1 a) $\lim_{x \to 2} \frac{x^2 + 7x + 3}{x - 7}$

$= \frac{(2)^2 + 7(2) + 3}{2 - 7}$

$= \frac{4 + 14 + 3}{-5}$

$= -\frac{21}{5}$

b) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$

$= \lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)}$

$= \lim_{x \to 3} (x+2)$

$= 3 + 2 = 5$

c) $\lim_{x \to 6} \frac{\sqrt{3+x} - 3}{x - 6} \frac{(\sqrt{3+x} + 3)}{(\sqrt{3+x} + 3)}$

$= \lim_{x \to 6} \frac{3 + x - 9}{(x-6)(\sqrt{3+x} + 3)}$

$= \lim_{x \to 6} \frac{(x-6)}{(x-6)(\sqrt{3+x} + 3)}$

$= \lim_{x \to 6} \frac{1}{\sqrt{3+x} + 3}$

$= \frac{1}{\sqrt{3+6} + 3} = \frac{1}{3+3} = \frac{1}{6}$

d) $\lim_{m \to 10} \frac{\frac{2}{m-4} - \frac{1}{3}}{m - 10} \frac{(3)(m-4)}{(3)(m-4)}$

$= \lim_{m \to 10} \frac{6 - (m-4)}{(m-10)(3)(m-4)}$

$= \lim_{m \to 10} \frac{6 - m + 4}{(m-10)(3)(m-4)}$

$= \lim_{m \to 10} \frac{(10 - m)}{(m-10)(3)(m-4)}$

$= \lim_{m \to 10} \frac{-1}{3(m-4)}$

$= \frac{-1}{3(10-4)} = \frac{-1}{18}$

e) $\lim_{x \to \infty} \frac{5 - 6x}{2x + 3} = \frac{-6}{2} = -3$

f) $\lim_{x \to \infty} \frac{3x^2}{x^3 - 4x} = 0$

g) $\lim_{x \to -\infty} \frac{x^2 + 5x}{x - 5} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to -\infty} \frac{x + 5}{1 - \frac{5}{x}} = \frac{-\infty + 5}{1} = -\infty$

h) $\lim_{x \to -\infty} \frac{\sqrt{4x^2 - x - 8}}{x + 9}$

$= \lim_{x \to -\infty} \frac{\sqrt{x^2(4 - \frac{1}{x} - \frac{8}{x^2})}}{x(1 + \frac{9}{x})}$

$= \lim_{x \to -\infty} \frac{2|x|}{x}$

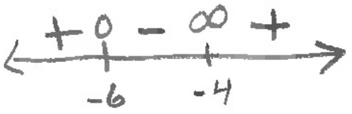
$= \lim_{x \to -\infty} \frac{-2x}{x}$

$= -2$

i) $\lim\limits_{x \to -4^-} \frac{x+6}{x+4} = -\infty$

- - 0 + + + + x+6 N

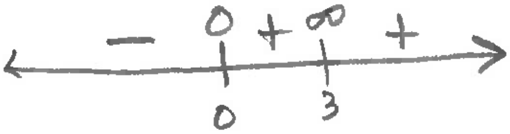
- - - - - ∞ + + x+4 D



j) $\lim\limits_{x \to 3^-} \frac{x}{(x-3)^2} = +\infty$

- - - - 0 + + + + x

+ + + + + ∞ + + $(x-3)^2$



k) $\lim\limits_{x \to 1} f(x)$

$\lim\limits_{x \to 1^+} x^3 - 2x^2$

$= (1)^3 - 2(1)^2 = -1$

$\lim\limits_{x \to 1^-} \frac{x+2}{x-2}$

$= \frac{1+2}{1-2} = \frac{3}{-1} = -3$

$\lim\limits_{x \to 1} f(x)$ DNE

2. $\lim\limits_{x \to -4} x^2 + x = \lim\limits_{x \to -4} kx + 10$

$(-4)^2 + (-4) = K(-4) + 10$

$16 - 4 = -4K + 10$

$12 = -4K + 10$

$2 = -4K$

$-\frac{1}{2} = K$

3. a) $f(x) = \frac{7}{x-4}$, at $x = 4$

i) $f(4) = \frac{7}{4-4}$

$= \frac{7}{0}$ ∴ $x = 4$ Vertical Asymptote

Infinite discontinuity at $x=4$

Breaks down i) $f(4)$ DNE

b) $f(x) = \begin{cases} x^2 + 2x, & \text{if } x \le 3 \\ 4x + 5, & \text{if } x > 3 \end{cases}$

i) $f(3) = (3)^2 + 2(3)$
$= 15$

ii) $\lim_{x \to 3} f(x)$

$\lim_{x \to 3^+} 4x + 5 = 4(3) + 5 = 17$ $\lim_{x \to 3^-} x^2 + 2x = (3)^2 + 2(3) = 15$

$\therefore \lim_{x \to 3}$ DNE

$\therefore$ Breaks down at ii)
Jump discontinuity at $x = 3$.

4. $x = -3$ removable discontinuity
iii) $\lim_{x \to -3} f(x) \neq f(-3)$

$x = -1$ removable discontinuity
i) $f(-1)$ DNE
$x = 1$ jump
ii) $\lim_{x \to 1} f(x)$ DNE

$x = 3$ infinite discontinuity
i) $f(3)$ DNE

5. HA
$\lim_{x \to \infty} \frac{2x^2 - 1}{4 - x^2}$, degrees same, ratio of leading coefficients
$y = -2$

VA
$4 - x^2 = 0$
$(2 - x)(2 + x) = 0$
$x = 2$ or $x = -2$

Derivatives

1. a) $f(x) = (2x+3)^2$

$f'(x) = 2(2x+3)^1 \cdot 2$

$= 4(2x+3)$

b) $f(x) = (x^3+x)(8-x)$

$f'(x) = (x^3+x)(-1) + (8-x)(3x^2+1)$

$= -x^3 - x + 24x^2 + 8 - 3x^3 - x$

$= -4x^3 + 24x^2 - 2x + 8$

$= -2(2x^3 - 12x^2 + x - 4)$

c) $f(x) = \left(\frac{2x}{x+2}\right)^{-2}$

$f(x) = \left(\frac{x+2}{2x}\right)^2$

$f'(x) = 2\left(\frac{x+2}{2x}\right)^1 \left[\frac{2x(1) - (x+2)(2)}{(2x)^2}\right]$

$= \frac{2(x+2)}{(2x)} \frac{(2x - 2x - 4)}{(4x^2)}$

$= \frac{2(x+2)(-4)}{8x^3} = \frac{-8(x+2)}{8x^3} = \frac{-(x+2)}{x^3}$

d) $y = \frac{\sqrt{x+3}}{x-2} = \frac{(x+3)^{1/2}}{x-2}$

$y' = \frac{(x-2)\frac{1}{2}(x+3)^{-1/2} - (x+3)^{1/2}(1)}{(x-2)^2}$

$= \frac{\frac{x-2}{2(x+3)^{1/2}} - (x+3)^{1/2}}{(x-2)^2} \left(\frac{2(x+3)^{1/2}}{2(x+3)^{1/2}}\right)$

$= \frac{x-2-2(x+3)}{2(x-2)^2(x+3)^{1/2}} = \frac{x-2-2x-6}{2(x-2)^2(x+3)^{1/2}} = \frac{-x-8}{2(x-2)^2(x+3)^{1/2}}$

$= \frac{-(x+8)}{2(x-2)^2(x+3)^{1/2}}$

e) $f(x) = (x+3)e^{3x}$

$$f'(x) = (x+3)e^{3x} \cdot 3 + e^{3x}(1)$$
$$= e^{3x}(3(x+3)+1)$$
$$= e^{3x}(3x+9+1)$$
$$= e^{3x}(3x+10)$$

f) $y = -6\sin(3x^2)$

$$y' = -6\cos(3x^2) \cdot 6x$$
$$y' = -36x\cos(3x^2)$$

g) $f(x) = \ln\left(\frac{x^2}{x-2}\right)$

$$f'(x) = \frac{1}{\left(\frac{x^2}{x-2}\right)}\left[\frac{(x-2)(2x) - x^2(1)}{(x-2)^2}\right] = \frac{(x-2)}{x^2}\left[\frac{2x^2-4x-x^2}{(x-2)^2}\right]$$
$$= \frac{\cancel{(x-2)}}{x^2}\left[\frac{x^2-4x}{(x-2)^{\cancel{2}}}\right]$$
$$= \frac{x(x-4)}{x^2(x-2)} = \frac{x-4}{x(x-2)}$$

h) $y = x^2 \tan 3x$

$$y' = x^2(\sec^2(3x)) \cdot 3 + (\tan 3x)(2x)$$
$$= 3x^2\sec^2(3x) + 2x\tan 3x$$
$$= x(3x\sec^2(3x) + 2\tan 3x)$$

i) $y = \cos^3(x^2+2)$

$$y' = 3[\cos(x^2+2)]^2 \cdot (-\sin(x^2+2))(2x)$$
$$y' = -6x\sin(x^2+2)\cos^2(x^2+2)$$

j) $y = 8^{x^3}$

$$y' = 8^{x^3} \cdot 3x^2 \cdot \ln 8$$

2. $y^2 + 3y - x = 4$

$2y\frac{dy}{dx} + 3\frac{dy}{dx} - 1 = 0$

$2y\frac{dy}{dx} + 3\frac{dy}{dx} = 1$

$\frac{dy}{dx}(2y+3) = 1$

$\frac{dy}{dx} = \frac{1}{2y+3}$

$\frac{dy}{dx} = (2y+3)^{-1}$

$\frac{d^2y}{dx^2} = -1(2y+3)^{-2} \cdot 2\frac{dy}{dx}$

$= \frac{-2}{(2y+3)^2} \cdot \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{-2}{(2y+3)^2} \cdot \left(\frac{1}{2y+3}\right)$

$\frac{d^2y}{dx^2} = \frac{-2}{(2y+3)^3}$

3. $f(x) = x^3 + 2x - 7$

$f(2) = (2)^3 + 2(2) - 7$

$f(2) = 5$

$f'(x) = 3x^2 + 2$

$f'(2) = 3(2)^2 + 2$

$f'(2) = 14$

$y - y_1 = m(x - x_1)$

$y - 5 = 14(x - 2)$

$y - 5 = 14x - 28$

$y = 14x - 23$

4. $f(x) = x^2 - 5x + 1$

$y = 5x - 10$
$m = 5$

$f'(x) = 2x - 5$

$2x - 5 = 5$

$2x = 10$

$x = 5$

$f(5) = (5)^2 - 5(5) + 1 = 1$

$(5, 1)$

5. $f(x) = (4x+3)^2$

$f'(x) = 2(4x+3) \cdot 4$

$f'(x) = 8(4x+3)$

$f'(1) = 8(4(1)+3)$

$f'(1) = 56$

$\therefore m_\perp = -\frac{1}{56}$

$(1, 49)$

$y - y_1 = m(x - x_1)$

$y - 49 = -\frac{1}{56}(x - 1)$

$y - 49 = -\frac{1}{56}x + \frac{1}{56}$

$56y - 2744 = -x + 1$

$x + 56y = 2745$

Application of Derivatives

1 a) $f(x) = x^2 + 6x - 8$

$f'(x) = 2x + 6$

$f'(x) = 0$

$2x + 6 = 0$

$2x = 6$

$x = -3$

$f'\cancel{(x)}\infty$

b) $f(x) = 5x^{1/5}$

$f'(x) = x^{-4/5}$

$f'(x) = \frac{1}{x^{4/5}}$

$f'(x) \cancel{=} 0$

$f'(x)\ \infty$

when $x^{4/5} = 0$

$\therefore\ x = 0$

c) $f(x) = \frac{x}{(x+1)^2}$

$f'(x) = \frac{(x+1)^2(1) - x(2(x+1))}{(x+1)^4}$ → divide out $(x+1)$

$= \frac{x+1 - 2x}{(x+1)^3}$

$= \frac{1-x}{(x+1)^3}$

$\underline{f'(x) = 0}$

$1 - x = 0$

$1 = x$

$\underline{f'(x)\ \infty}$

$x = -1$ → this is an undefined value in the function.

2. a) $f(x) = 4 - x^2$, $[-1, 2]$

$f'(x) = -2x$

$\underline{f'(x) = 0}$

$-2x = 0$

$x = 0$

$f'\cancel{(x)}\infty$

$f(-1) = 4 - (-1)^2 = 3$

$f(0) = 4 - (0)^2 = 4$ abs max 4

$f(2) = 4 - (2)^2 = 0$ abs min 0

b) $f(x) = \frac{27}{x^2+9}$, $[-2, 3]$

$f(x) = 27(x^2+9)^{-1}$

$f'(x) = -27(x^2+9)^{-2} \cdot 2x$

$f'(x) = \frac{-54x}{(x^2+9)^2}$

$\underline{f'(x) = 0}$

$-54x = 0$

$x = 0$

$f'\cancel{(x)}\infty$

$f(-2) = \frac{27}{(-2)^2+9} = \frac{27}{13}$

$f(0) = \frac{27}{0^2+9} = 3$

$f(3) = \frac{27}{3^2+9} = \frac{27}{18} = \frac{3}{2}$

Abs max 3

Abs min $\frac{3}{2}$

c) $f(x) = x\sqrt{3-x}$, $[-1, 3]$

$f'(x) = x \cdot \frac{1}{2}(3-x)^{-1/2} \cdot (-1) + (3-x)^{1/2}$

$= \frac{-x}{2(3-x)^{1/2}} + \frac{(3-x)^{1/2} \cdot 2(3-x)^{1/2}}{2(3-x)^{1/2}}$

$= \frac{-x}{2(3-x)^{1/2}} + \frac{2(3-x)}{2(3-x)^{1/2}}$

$= \frac{-x+6-2x}{2(3-x)^{1/2}} = \frac{6-3x}{2(3-x)^{1/2}}$

$\underline{f'(x) = 0}$

$6-3x = 0$

$x = 2$

$\underline{f'(x)\ \infty}$

$2(3-x)^{1/2} = 0$

$x = 3$ → (endpoint of interval)

$f(-1) = -1(\sqrt{3-(-1)}) = -2$

$f(2) = 2(\sqrt{3-(2)}) = 2$

$f(3) = 3(\sqrt{3-3}) = 0$

Abs Max $= 2$

Abs Min $= -2$

3. $f(x) = x^3+2x^2-x$ $[-1,2]$

$f'(x) = \frac{f(2)-f(-1)}{2-(-1)}$

$3x^2+4x-1 = \frac{14-2}{3}$

$3x^2+4x-1 = 4$

$3x^2+4x-5=0$

$x = \frac{-4 \pm \sqrt{(4)^2-4(3)(-5)}}{2(3)}$

$x = \frac{-4 \pm \sqrt{16+60}}{6}$ $\to$ $x = \frac{-4 \pm 8.718}{6}$

$x = \frac{-4+8.718}{6}$ or $x = \frac{-4-8.718}{6}$

$\boxed{x = .786}$ or $x = \cancel{-2.120}$

only one to fit in interval.

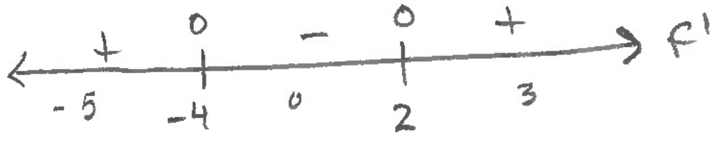
4. a) $f(x) = \frac{1}{3}x^3+x^2-8x-2$

$f'(x) = x^2+2x-8$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4$ or $x=2$



inc

$(-\infty,-4) \cup (2,\infty)$

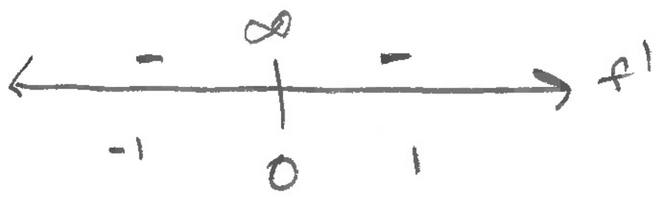
dec

$(-4,2)$

b) $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = \frac{-1}{x^2}$

$\frac{f'(x)\ \infty}{x=0}$



dec

$(-\infty,0) \cup (0,\infty)$

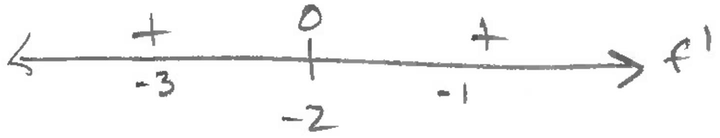
5 a) $f(x) = (x+2)^3$

$f'(x) = 3(x+2)^2$

$\underline{f'(x) = 0}$

$3(x+2)^2 = 0$

$x = -2$



$\underline{\text{inc}}$

$(-\infty, -2) \cup (-2, \infty)$

No rel extrema

b) $f(x) = x^3 + 6x^2 - 15x - 90$

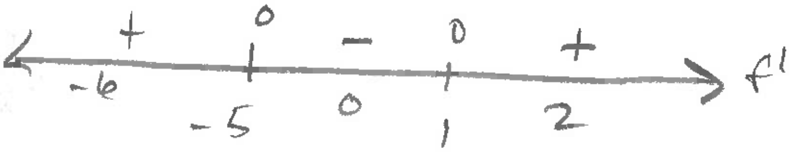
$f'(x) = 3x^2 + 12x - 15$

$3x^2 + 12x - 15 = 0$

$3(x^2 + 4x - 5) = 0$

$3(x+5)(x-1) = 0$

$x = -5$ or $x = 1$



$\underline{\text{inc}}$

$(-\infty, -5) \cup (1, \infty)$

$\underline{\text{dec}}$

$(-5, 1)$

$\underline{\text{rel max}}$

$f(-5) = 10$

$\underline{\text{rel min}}$

$f(1) = -98$

6a) $f(x) = x^3 - 3x^2$

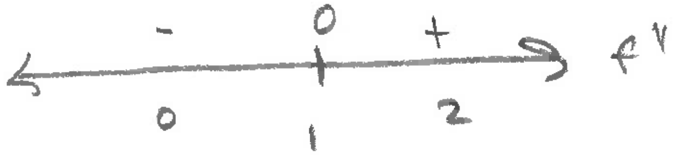
$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

$\underline{f''(x) = 0}$

$6x - 6 = 0$

$x = 1$



$\underline{\text{CU}}$

$(1, \infty)$

$\underline{\text{CD}}$

$(-\infty, 1)$

b) $f(x) = x^4 + 6x^3$

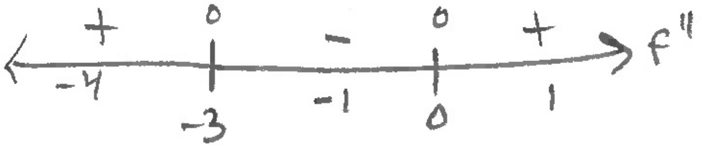
$f'(x) = 4x^3 + 18x^2$

$f''(x) = 12x^2 + 36x$

$\underline{f''(x) = 0}$ $12x^2 + 36x = 0$

$12x(x+3) = 0$

$x = 0$ or $x = -3$



$\underline{\text{CU}}$

$(-\infty, -3) \cup (0, \infty)$

$\underline{\text{CD}}$

$(-3, 0)$

6c) $f(x) = \frac{4}{x^2+3} = 4(x^2+3)^{-1}$

$f'(x) = -4(x^2+3)^{-2} \cdot 2x$

$f'(x) = \frac{-8x}{(x^2+3)^2}$

$f''(x) = \frac{(x^2+3)^2(-8) - (-8x)(2(x^2+3)^1 \cdot 2x)}{(x^2+3)^4}$

$f''(x) = \frac{-8(x^2+3) + 32x^2}{(x^2+3)^3}$

$f''(x) = \frac{-8x^2 - 24 + 32x^2}{(x^2+3)^3}$

$f''(x) = \frac{24x^2 - 24}{(x^2+3)^3}$

$\underline{f''(x) = 0}$

$24x^2 - 24 = 0$

$24(x^2-1) = 0$

$24(x-1)(x+1) = 0$

$x = 1$ or $x = -1$

$\underline{CU}$

$(-\infty, -1) \cup (1, \infty)$

$\underline{CD}$

$(-1, 1)$

7. $f(x) = x^3 + 3x^2 - 24x$

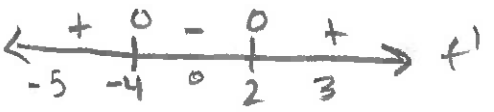
a) $f'(x) = 3x^2 + 6x - 24$

$3x^2 + 6x - 24 = 0$

$3(x^2 + 2x - 8) = 0$

$3(x+4)(x-2) = 0$

$x = -4$ or $x = 2$



b) $\underline{\text{inc}}$

$(-\infty, -4) \cup (2, \infty)$

dec

$(-4, 2)$

c) rel max

$f(-4) = 80$

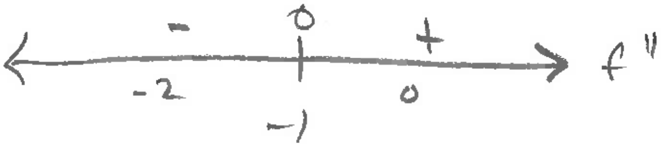
rel min

$f(2) = -28$

d) $f''(x) = 6x + 6$

$6x + 6 = 0$

$x = -1$



e) $\underline{\text{CD}}$ $(-\infty, -1)$

$\underline{\text{CU}}$ $(-1, \infty)$

f) IP

$f(-1) = 26$

g) $\underline{\text{y int}}$

let $x = 0$

$y = 0$

$(0, 0)$

$\underline{\text{x int}}$

let $y = 0$

$0 = x^3 + 3x^2 - 24x$

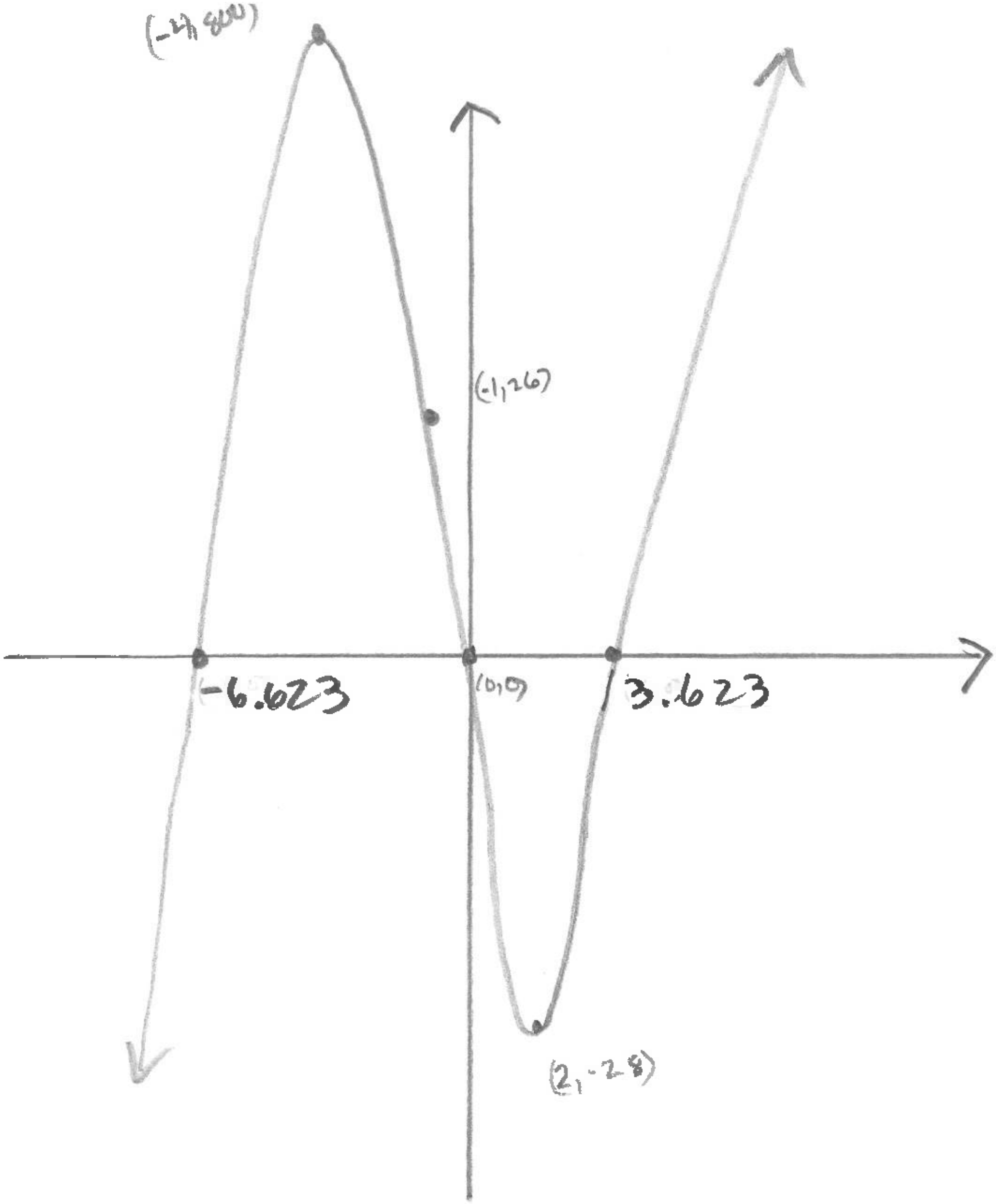
$0 = x(x^2 + 3x - 24)$

$0 = x$ or $x^2 + 3x - 24 = 0$ — quad formula

$x = 3.623$

$x = -6.623$

h)



8. Let $x=$ first # $y=2^{nd}$ #

$x+y=4$

$y=4-x$

$P=xy$

$P=x(4-x)$

$P=4x-x^2$

$P'=4-2x$

$\underline{P'=0}$

$4-2x=0$

$4=2x$

$2=x$

Proof

$P''=-2$

∴ CD

∴ $x=2$ max

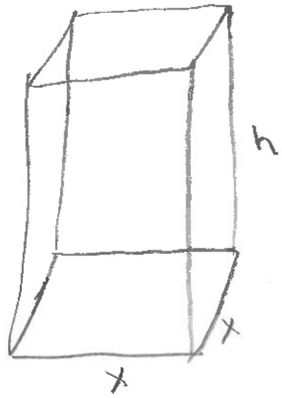
Find y

$y=4-2$

$y=2$

∴ The numbers are 2 and 2 and max product is 4.

9.



Let $x=$ width, length $h=$ height

$V=x^2h$

$2700=x^2+4xh$

$\frac{2700-x^2}{4x}=h$

$V=x^2\left[\frac{2700-x^2}{4x}\right]$

$V=\frac{2700x-x^3}{4}$

$V=675x-\frac{1}{4}x^3$

$V'=675-\frac{3}{4}x^2$

$\underline{V'=0}$

$675-\frac{3}{4}x^2=0$

$675=\frac{3}{4}x^2$

$2700=3x^2$

$900=x^2$

$\pm30=x$

$x=30$ cm

Proof

$V''=-\frac{6}{4}x$

$V''(30)=-\frac{6}{4}(30)<0$

∴ CD

∴ $x=30$ max

Find h

$h=\frac{2700-x^2}{4x}$

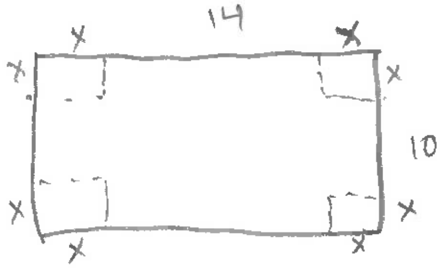
$h=\frac{2700-(30)^2}{4(30)}$

$h=\frac{2700-900}{120}$

$h=15$ cm

∴ The dimensions are 30cm x 30cm x 15cm.

10.



Let x = length of cut

$V = x(14-2x)(10-2x)$

$V = x(140-28x-20x+4x^2)$

$V = 4x^3-48x^2+140x$

$V' = 12x^2-96x+140$

$\underline{V' = 0}$

$$x = \frac{-(-96) \pm \sqrt{(-96)^2-4(12)(140)}}{2(12)}$$

$$x = \frac{96 \pm \sqrt{2496}}{24}$$

$$x = \frac{96 \pm 49.960}{24}$$

$x = \frac{96+49.960}{24}$ or $x = \frac{96-49.960}{24}$

$x = 6.\cancel{082}$ in or $x = 1.918$ in

too big

Proof

$V'' = 24x-96$

$V''(1.918) = 24(1.918)-96 < 0$

$\therefore$ CD

$\therefore$ $x = 1.918$ max.

The length of cut should be 1.918 in.

11.

$\frac{dV}{dt} = 18 cm^3/s$

$r = 15 cm$

$h_c = 25 cm$

$\frac{dh}{dt} = ?$

$h_w = 20cm$

$\frac{r}{h} = \frac{15}{25}$

$r = \frac{3h}{5}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{3h}{5}\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{9h^2}{25}\right) h$

$V = \frac{9\pi h^3}{75}$

$\frac{dV}{dt} = \frac{27\pi h^2}{75} \frac{dh}{dt}$

$$\frac{\left(\frac{dV}{dt}\right)}{\left(\frac{27\pi h^2}{75}\right)} = \frac{dh}{dt}$$

$$\frac{18cm^3/s}{\left(\frac{27\pi(20cm)^2}{75}\right)} = \frac{dh}{dt}$$

$0.040 cm/s = dh/dt$

The water level is rising at a rate of 0.040 cm/s.

12.

$\frac{dV}{dt} = 5m^3/min$

$V = \pi r^2 h$

since r is constant sub 3m in for r.

$V = (9m^2)\pi h$

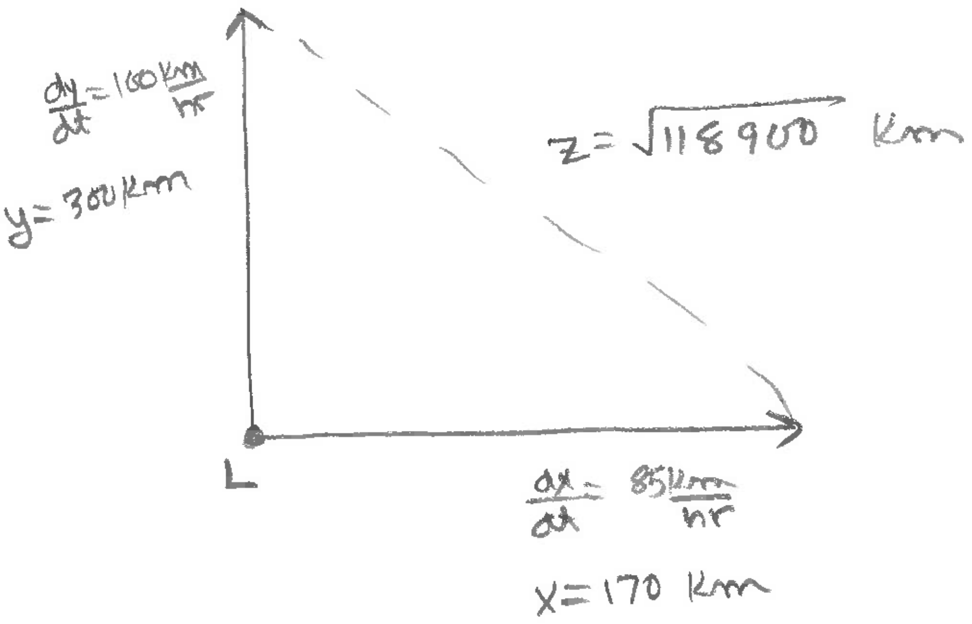
$\frac{dV}{dt} = (9m^2)\pi \frac{dh}{dt}$

$5\frac{m^3}{min} = (9m^2)\pi \frac{dh}{dt}$

$.177\frac{m}{min} = \frac{5}{9\pi} m/min = \frac{dh}{dt}$

The tank is rising at a rate of .177 m/min.

13.



$$x^2+y^2=z^2$$
$$(170)^2+(300)^2=z^2$$
$$\sqrt{118900}\text{ km}=z$$

$$x^2+y^2=z^2$$
$$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}z\frac{dz}{dt}$$
$$\frac{x\frac{dx}{dt}+y\frac{dy}{dt}}{z}=\frac{dz}{dt}$$
$$\frac{(170\text{ km})(85\frac{\text{km}}{\text{hr}})+(300\text{ km})(100\frac{\text{km}}{\text{hr}})}{\sqrt{118900}\text{ km}}=\frac{d}{d}$$
$$128.908\frac{\text{km}}{\text{hr}}=\frac{dz}{dt}$$

The distance between the planes is increasing at a rate of 128.908 km/hr.

14. $s(t) = 2t^3 - 7t^2 + 3t$

a) $s(3) = 2(3)^3 - 7(3)^2 + 3(3)$

$s(3) = 0$ m

b) $v(t) = s'(t) = 6t^2 - 14t + 3$

c) $v(2) = 6(2)^2 - 14(2) + 3$

$v(2) = -1$ m/s

d) ave vel $= \dfrac{s(6) - s(4)}{6-4}$

$= \dfrac{198 - 28}{2}$

$= 85$ m/s

e) $a(t) = v'(t) = 12t - 14$

f) $a(10) = 12(10) - 14 = 106$ m/s^2

g) set $a(t) = 70$

$12t - 14 = 70$

$12t = 84$

$t = 7$ s

$v(7) = 199$ m/s

Definite Integrals and Indefinite Integrals

1.
$$\int_1^{15} f(x)\,dx \cong 2(5)+(2)(5.4)+(3)(6.2)+(1)(8.1)+(4)(8.9)+2(12.9)$$
$$\cong 10+10.8+18.6+8.1+35.6+25.8$$
$$\cong 108.9$$

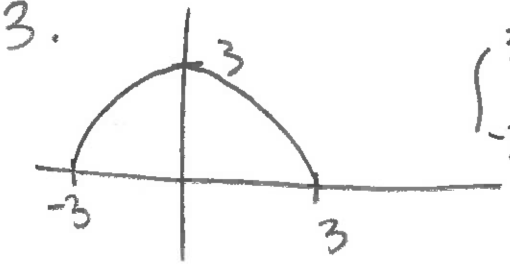
2. a) $\int_0^{24} R(t)\,dt \cong 6\left[9.3+6.2+10.4+7.5\right] \cong 200.4$ gallons.

b) Represents # of gallons to flow through pipe in $t=0$ to $t=24$ hours.

c) $R'(13) \approx \dfrac{R(15)-R(12)}{15-12}$
$= \dfrac{10.4-9.1}{3} = 0.433$ gallons/hr^2

d) $\frac{1}{24}\int_0^{24} R(t)\,dt = \dfrac{200.4}{24} = 8.350$ g/h
Represents average rate flow of water in gallons from $t=0$ to $t=24$h.

3.



$$\int_{-3}^{3}\sqrt{9-x^2}\,dx = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$$

4. a) $\int_0^4 x^3\,dx = \dfrac{x^4}{4}\Big|_0^4 = \dfrac{(4)^4}{4} - \dfrac{(0)^4}{4} = 64$

b) $\int_{-2}^{3}(x+1)^2\,dx = \int_{-2}^{3}(x^2+2x+1)\,dx$
$$= \frac{x^3}{3}+x^2+x\Big|_{-2}^{3}$$
$$= \left(\frac{(3)^3}{3}+(3)^2+3\right) - \left(\frac{(-2)^3}{3}+(-2)^2+(-2)\right)$$
$$= (9+9+3) - \left(-\frac{8}{3}+4-2\right)$$
$$= 21 - \left(2-\frac{8}{3}\right) = 21-\left(-\tfrac{2}{3}\right) = \frac{63}{3}+\frac{2}{3} = \frac{65}{3}$$

c) $\int_{\pi/2}^{\pi} \sin x\,dx = -\cos x\Big|_{\pi/2}^{\pi} = -\left[\cos\pi - \cos\pi/2\right]$
$= -\left[-1-0\right] = 1$

d) $\int_3^5 \frac{1}{x^2}\,dx = \int_3^5 x^{-2}\,dx = -x^{-1}\Big|_3^5 = \frac{-1}{x}\Big|_3^5 = \frac{-1}{5} - \left(\frac{-1}{3}\right)$

$= \frac{-1}{5} + \frac{1}{3}$

$= \frac{-3}{15} + \frac{5}{15} = \frac{2}{15}$

e) $\int_2^5 \frac{1}{x+3}\,dx = \ln|x+3|\,\Big|_2^5 = \ln|5+3| - \ln|2+3|$

$= \ln 8 - \ln 5 = \ln(8/5)$

f) $\int_{\ln 2}^{\ln 5} e^x\,dx = e^x\Big|_{\ln 2}^{\ln 5} = e^{\ln 5} - e^{\ln 2} = 5 - 2 = 3.$

g) $\int_{-5}^{5} (5x^4 + 6x)\,dx = x^5 + 3x^2\Big|_{-5}^{5}$

$= \left[(5)^5 + 3(5)^2\right] - \left[(-5)^5 + 3(-5)^2\right]$

$= [3125 + 75] - [-3125 + 75]$

$= 6250$

h) $\int_1^9 \sqrt{x}\,dx = \int_1^9 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big|_1^9 = \frac{2}{3}\left[9^{3/2} - 1^{3/2}\right]$

$= \frac{2}{3}[27 - 1]$

$= \frac{2}{3}(26) = \frac{52}{3}$

5. a) $g(4) = \int_0^4 f(t)\,dt$

$= -(1)(3) - \frac{1}{2}(1)(1)$

$= -\frac{7}{2}$

b) $g(6) = \int_0^6 f(t)\,dt$

$= -(1)(3) - \frac{1}{2}(1)(1) + \frac{1}{2}(2)(2)$

$= -\frac{7}{2} + 2 = -\frac{3}{2}$

c) $g'(x) = f(x)$

$g'(6) = f(6) = 2$

d) $g''(8) = f'(8)$

slope at $x = 8$

$= -\frac{1}{2}$

e) $x = 6$ as g' Δ's inc to dec.

f) point $(6, -3/2)$ $m = g'(6) = 2$

$y + \frac{3}{2} = 2(x-6)$

$y = 2x - 12 - \frac{3}{2}$ $y = 2x - \frac{27}{2}$

6 a) $\int_{-4}^{-2} f' = f(-2) - f(-4)$

$-\frac{1}{2}(2)(1) = 4 - f(-4)$

$f(-4) = 4 + 1 = 5$

b) $\int_{-2}^{4} f' = f(4) - f(-2)$

$-\frac{\pi}{2}(2)^2 + \frac{1}{2}(2)(2) = f(4) - 4$

$6 - 2\pi = f(4)$

c) $\int_{-2}^{6} f' = f(6) - f(-2)$

$-\frac{\pi}{2}(2)^2 + \frac{1}{2}(4)(2) = f(6) - 4$

$8 - 2\pi = f(6)$

d)

x	$f(x)$
-4	5
2	$4-2\pi$
6	$8-2\pi$

↗ abs min

$\int_{-2}^{2} f' = f(2) - f(-2)$

$-\frac{\pi}{2}(2)^2 = f(2) - 4$

$4 - 2\pi = f(2)$

e) $x = -2$ f' Δ's inc to dec

$x = 4$

$x = 0$ f' Δ's dec to inc

f) $(-2, 0)$ $f' < 0$ and f' dec

7. a) $\int \left(1 - x^{-2} + x^{-3} - x^{-4}\right) dx$

$$x + x^{-1} - \frac{x^{-2}}{2} + \frac{x^{-3}}{3} + c$$

b) $\int 6\,dx = 6x + c$

c) $\int \frac{4}{x}\,dx = 4\ln|x| + c$

d) $\int -2x^{-3/2}\,dx$

$$= -2\left[\frac{x^{-1/2}}{-1/2}\right] + c$$

$$= 4x^{-1/2} + c$$

e) $\int e^{11x}\,dx = \frac{e^{11x}}{11} + c$

f) $\int (6x - \sin x + \cos x)\,dx$

$$= 3x^2 + \cos x + \sin x + c$$

g) $\int (x+1)^2\,dx = \int (x^2 + 2x + 1)\,dx = \frac{x^3}{3} + x^2 + x + c$

h) $\int \frac{2x - x^2}{\sqrt{x}}\,dx = \int \left(2x^{1/2} - x^{3/2}\right) dx$

$$= 2\left[\frac{x^{3/2}}{3/2}\right] - \frac{x^{5/2}}{5/2} + c$$

$$= \frac{4}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$$

i) $\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + c$

8. a) $\int (3x-8)^6 \, dx$

let $u = 3x-8$

$du = 3\,dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3} \int u^6 \, du$

$= \frac{1}{3} \left[\frac{u^7}{7} \right] + C$

$= \frac{1}{21} u^7 + C$

$= \frac{1}{21} (3x-8)^7 + C$

b) $\int x^2 (2x^3+3)^{14} \, dx$

let $u = 2x^3 + 3$

$du = 6x^2 \, dx$

$\frac{1}{6} du = x^2 \, dx$

$= \frac{1}{6} \int u^{14} \, du$

$= \frac{1}{6} \left[\frac{u^{15}}{15} \right] + C$

$= \frac{1}{90} u^{15} + C$

$= \frac{1}{90} (2x^3+3)^{15} + C$

c) $\int x^2 \cos x^3 \, dx$

let $u = x^3$

$du = 3x^2 \, dx$

$\frac{1}{3} du = x^2 \, dx$

$= \frac{1}{3} \int \cos u \, du$

$= \frac{1}{3} \sin u + C$

$= \frac{1}{3} \sin x^3 + C$

d) $\int (x+5) \, e^{x^2+10x} \, dx$

let $u = x^2 + 10x$

$du = (2x + 10) \, dx$

$\frac{1}{2} du = (x+5) \, dx$

$= \frac{1}{2} \int e^u \, du$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2+10x} + C$

e) $\int \sqrt{e^x} \, dx = \int (e^x)^{1/2} \, dx = \int e^{\frac{1}{2}x} \, dx$

let $u = \frac{1}{2}x$

$du = \frac{1}{2} dx$

$2\,du = dx$

$= 2 \int e^u \, du$

$= 2e^u + C = 2e^{\frac{1}{2}x} + C$

Applications of Integration

1. $$\int_{-1}^{2} (x^2-2x+3) - (-1-x^2)\, dx$$

$$= \int_{-1}^{2} (x^2 - 2x + 3 + 1 + x^2)\, dx = \int_{-1}^{2} (2x^2 - 2x + 4)\, dx$$

$$= \frac{2}{3}x^3 - x^2 + 4x \Big|_{-1}^{2}$$

$$= \frac{2}{3}(2)^3 - (2)^2 + 4(2) - \left(\frac{2}{3}(-1)^3 - (-1)^2 + 4(-1)\right)$$

$$= \frac{16}{3} - 4 + 8 - \left(\frac{2}{3} - 1 - 4\right)$$

$$= \frac{16}{3} + 4 + \frac{2}{3} + 5 = 9 + \frac{18}{3} = 9 + 6 = 15$$

2. $f(x) = x-4$ $\quad g(x) = x^3 - 4x^2$

Find intersection points:

$$x-4 = x^3 - 4x^2$$

$$0 = x^3 - 4x^2 - x + 4$$

By grouping factor

$$0 = x^2(x-4) - 1(x-4)$$

$$0 = (x^2-1)(x-4)$$

$$0 = (x-1)(x+1)(x-4)$$

$x = 1 \quad x = -1 \quad x = 4$

Region I

$$\int_{-1}^{1} x^3 - 4x^2 - (x-4)\, dx$$

$$\int_{-1}^{1} (x^3 - 4x^2 - x + 4)\, dx$$

$$= \frac{x^4}{4} - \frac{4x^3}{3} - \frac{x^2}{2} + 4x \Big|_{-1}^{1}$$

$$= \frac{(1)^4}{4} - \frac{4(1)^3}{3} - \frac{(1)^2}{2} + 4(1) - \left[\frac{(-1)^4}{4} - \frac{4(-1)^3}{3} - \frac{(-1)^2}{2} + 4(\right.$$

$$= \left(\frac{1}{4} - \frac{4}{3} - \frac{1}{2} + 4\right) - \left(\frac{1}{4} + \frac{4}{3} - \frac{1}{2} - 4\right)$$

$$= \frac{29}{12} - \left(-\frac{35}{12}\right)$$

$$= \frac{64}{12} = \frac{16}{3}$$

Region II

$$\int_1^4 (x-4) - (x^3-4x^2)\,dx$$

$$= \int_1^4 (x-4-x^3+4x^2)\,dx$$

$$= \frac{x^2}{2} - 4x - \frac{x^4}{4} + \frac{4}{3}x^3 \Big|_1^4$$

$$= \frac{(4)^2}{2} - 4(4) - \frac{(4)^4}{4} + \frac{4}{3}(4)^3 - \left(\frac{(1)^2}{2} - 4(1) - \frac{(1)^4}{4} + \frac{4}{3}(1)^3\right)$$

$$= 8 - 16 - 64 + \frac{256}{3} - \left(\frac{1}{2} - 4 - \frac{1}{4} + \frac{4}{3}\right)$$

$$= -72 + \frac{256}{3} - \left(\frac{-29}{12}\right)$$

$$= \frac{63}{4}$$

$\therefore$ Total Area

$$\frac{16}{3} + \frac{63}{4} = \frac{253}{12}$$

3. $$\int_m^4 4x\,dx = -18$$

$$2x^2 \Big|_m^4 = -18$$

$$2(4)^2 - 2(m)^2 = -18$$

$$32 - 2m^2 = -18$$

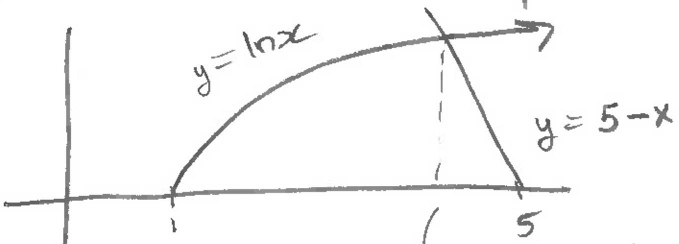
$$50 = 2m^2$$

$$25 = m^2$$

$$\pm 5 = m$$

$$\boxed{m = -5}$$

4. Find intersection point with calculator.



$\left(\underset{A}{3.6934414},\ \underset{B}{1.3065586}\right)$

a) Area = $\int_1^A (\ln x)\,dx + \int_A^5 (5-x)\,dx$

$= 2.986$

b) $\int_0^K \left((5-y)-(e^y)\right)dy = \frac{1}{2}(2.986)$